

Health care provider profiling: Fixing observation period or fixing sample size?

Supplementary Material: Details of statistical computations

Basic notation

Let Y_{ji} denote the binary outcome of patient i at provider j , and let $p_j = P(Y_{ji} = 1)$ denote the true probability of the event of interest at provider j . With \hat{p}_j we denote the relative frequency of events at provider j , and with \bar{p} the overall relative frequency over all providers.

Confidence intervals, control limits, and statistical significance

For the construction of confidence intervals for p_j the Wilson score approach is used. Control limits are based on the normal approximation to the binomial distribution. Statistical significance is assessed by comparing the lower bound of the 95% confidence interval with \bar{p} or $\bar{p} + \Delta^*$, respectively.

Estimation of the inter-provider variation and computation of posterior means and posterior standard deviations

The logistic random intercept model

$$\text{logit } P(Y_{ji} = 1) = \mu + \delta_j \quad \text{with } \delta_j \sim N(0, \sigma^2)$$

is fitted using the ML principle as implemented in the `melogit` command of Stata. The command provides also estimates ν_j and τ_j for the mean and standard deviation of the posterior distribution of δ_j .

Computation of posterior probabilities

For a given value Δ , the following approximation is used to compute the posterior probability

$$\begin{aligned} P(\Delta_j > \Delta) &= P(p_j > \bar{p} + \Delta) \\ &= P(\text{logit } p_j > \text{logit}(\bar{p} + \Delta)) \\ &\approx P(\text{logit } p_j - \mu > \text{logit}(\bar{p} + \Delta) - \text{logit } \bar{p}) \\ &= P(\delta_j > \text{logit}(\bar{p} + \Delta) - \text{logit } \bar{p}) \\ &\approx 1 - \Phi\left(\frac{\text{logit}(\bar{p} + \Delta) - \text{logit } \bar{p} - \nu_j}{\tau_j}\right) \end{aligned}$$

In the special case $\Delta = 0$ we obtain

$$P(\Delta_j > 0) \approx 1 - \Phi((0 - \nu_j)/\tau_j) = \Phi(\nu_j/\tau_j)$$

Simulation study

As the true probabilities p_j are drawn from a normal distribution, it can happen (with a probability of 0.0038) that negative values are drawn. These negative values are replaced by the value 0.

Statistical software

All computations are performed with Stata 16.1.